Math 3450 - Test 1

Name:

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T	

1. [3 points] List 3 elements from the set $T = \{x^2 + x - 1 \mid x \in \mathbb{Z}\}$.

2. [15 points - 3 each] Let $A = \{-1, 0, 5, 1, 10, 3\}$, $B = \{1, 10, 3, \pi, \frac{1}{2}\}$, $C = \{0, 5, \pi\}$ and $D = \{1, 5\}$. Compute the following.

   (a) $A \cup B$

   (b) $B - C$.

   (c) $(A \cap C) \cup (A - C)$

   (d) $C \times D$

   (e) The power set $\mathcal{P}(C)$.

3. [9 points - 3 each]

(a) <u>True or False?</u> Explain why.

$$27 \equiv -33 \pmod 5$$

(b) <u>True or False?</u> Explain why.

$$\overline{-6} = \overline{27} \quad \text{in } \mathbb{Z}_8$$

(c) In $\mathbb{Z}_5 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}\}$, calculate

$$\overline{a} = \overline{2} \cdot \overline{3} + \overline{4} \cdot \overline{4}$$

Reduce $\overline{a}$ so that a satisfies $0 \le a \le 4$.

4. [10 points] Let $A_n = \{-2n, 0, 2n\}$.

(a) List the elements in each of the sets A_1, A_2 and A_3.

(b) Calculate $\displaystyle\bigcap_{n=1}^{\infty} A_n$ and $\displaystyle\bigcup_{n=1}^{\infty} A_n$.

5. [10 points]

Pick ONE of the following to prove. Only pick one. If you do both then I will grade (A).

A) Let X and Y be sets. Prove that $\mathcal{P}(X \cap Y) = \mathcal{P}(X) \cap \mathcal{P}(Y)$.

B) Let $S = \mathbb{N} \times \mathbb{N}$. Define the relation $\sim$ on S where $(a, b) \sim (c, d)$ if and only if $a + d = b + c$. Prove that $\sim$ is an equivalence relation on S.

6. [10 points] Let A, B, and C be sets. Prove that
$$A \times (B - C) \subseteq (A \times B) - (A \times C).$$

7. [10 points] Consider the set of integers $\mathbb{Z}$. Let $n \in \mathbb{Z}$ with $n \geq 2$. Given $a, b \in \mathbb{Z}$, define $a \sim b$ if and only if n divides $a - b$. Prove that $\sim$ is an equivalence relation.